NOLTR 66-117

AD 649255

MONTE CARLO INVESTIGATIONS OF SMALL SAMPLE BRUCETON TESTS

17 FEBRUARY 1967

NOL

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

NOLTR 66-117

Distribution of this document is unlimited.

MONTE CARLO INVESTIGATIONS OF SMALL SAMPLE BRUCETON TESTS

By

L. D. Hampton

ABSTRACT: Monte Carlo investigations of Bruceton tests (twenty-five and fifty trials) show the following characteristics: (1) a bias in the estimate of the standard deviation causes predictions of reliability or safety based on such tests to be too optimistic; (2) there are additional biases (in the parameters G and H) which will cause underestimates of the errors of the mean and standard deviation; (3) there is little correlation between the actual standard deviation and its estimate as obtained from the short Bruceton test; (4) the order in which the items of the sample are tested has a serious effect upon the estimates obtained; (5) a poor choice of starting level can give misleading results when the step is small.

EXPLOSION DYNAMICS DIVISION
EXPLOSIONS RESEARCH DEPARTMENT
U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

17 February 1967

MONTE CARLO INVESTIGATIONS OF SMALL SAMPLE BRUCETON TESTS

This report gives the results of Monte Carlo investigations of Bruceton tests with a limited number of items. Limited-sample size Bruceton testing is all too prevalent a practice. Many experimenters do not realize the serious errors that can result from using samples that are too small. This report has been written to identify and quantify some of the types of errors that are to be expected so that the experimenter can have a better idea of the consequences when he is forced to use less than desirable-sized samples.

This work was carried out under Task NOL 443/NWL. The results should be of value to those who have occasion to design experiments and/or make reliability or safety predictions based on Bruceton tests of fifty or fewer items.

E. F. SCHREITER
Captain, USN
Commander

C J Aronson

C. J. ARONSON
By direction

TABLE OF CONTENTS

	Page
INTRODUCTION	1
APPROACH	1
BIAS IN ESTIMATION OF STANDARD DEVIATION	3
VALUES OF G AND H	4
CORRELATION OF BRUCETON s WITH SAMPLE s	6
VARIATION IN ESTIMATES OF MEAN AND STANDARD DEVIATION AS A RESULT OF ORDER OF TESTING	7
STARTING LEVEL AND STEP SIZE CONSIDERATIONS	8
SUMMARY	9
REFERENCES	11

TABLES

Number	Title	Page
1	RESULTS OF FIFTY MONTE CARLO BRUCETON TESTS, 25 TRIALS	12
2	RESULTS OF SIXTY-THREE MONTE CARLO BRUCETON TESTS, 200 TRIALS	14
3	CORRELATION OF BRUCETON s WITH SAMPLE s	16
4	TWENTY-FIVE TRIAL SELECTION ORDER EXPERIMENTS	17
5	ONE-HUNDRED TRIAL SELECTION ORDER EXPERIMENTS	18
6	TWO-HUNDRED TRIAL SELECTION ORDER EXPERIMENTS	19
7	STANDARD DEVIATIONS AND THEIR CORRECTION TO ACCOUNT FOR SELECTION ORDER EFFECT	20
8	EFFECT OF POOR CHOICE OF STARTING LEVEL ON ESTIMATED PARAMETERS	20
9	INDIVIDUAL TEST RESULTS FOR INVESTIGATION OF EFFECT OF STARTING LEVEL	21

ILLUSTRATIONS

Figure	Title	Page
1	BRUCETON TEST NO. 1	25
2	BRUCETON TEST NO. 2	25
3	BRUCETON TEST NO. 3	26
4	BRUCETON TEST NO. 4	26
5	VARIABILITY OF ESTIMATES OF 90, 50, AND 10% POINTS BASED ON 20-SHOT BRUCETON SAMPLES FROM A KNOWN POPULATION	27

INTRODUCTION

The Bruceton method of testing was designed for use in explosive sensitivity experiments to determine the fifty per cent response points. The results are analyzed by a method due to the Applied Mathematics Panel of the National Defense Research Council[1]*. By assuming an infinite sample they developed formulas for estimating the mean and standard deviation and showed how to set confidence limits on these estimates. As they pointed out , the assumption of infinite sample size limits the use of their method to large samples. They suggest that samples of one hundred would do very well for most purposes and that their analysis should not be used for samples of less than fifty. In spite of the authors' stated limitations, the Bruceton test with small sample size is now being extensively used to make estimates of remote sensitivity points, i.e., 99.9 and 0.01 per cent. From these estimates safety and reliability predictions often are being made for explosive ordnance devices. This report gives the results of a study of four features of the Bruceton test when small sample sizes are used. They are: first, bias in the estimation of the standard deviation; second, the correlation between the sample parameters, mean and standard deviation, and their estimates m and s obtained from the Bruceton test; third, variations in the estimates for the mean and standard deviation of a single sample due to different order of testing of the individual items; and fourth, the results of a poor choice of starting level on the estimates of mean and standard deviation. Monte Carlo investigations were used to obtain the estimates of m and s when a small sample size is used.

APPROACH

In these Monte Carlo investigations the sensitivities (critical levels) of simulated explosive devices under test were represented by random numbers having a normal distribution with known mean and standard deviation. These numbers were generated on a high speed computer using the scheme of Reference 2. A set of random numbers is formed having a uniform distribution between zero and one. These numbers are then transformed to a

---

*References may be found on page 11.

normal distribution with the desired mean and standard deviation. A check of ten thousand of these numbers showed no significant departure from the expected normal distribution.

We next set up a series of trial levels equally spaced with respect to the stimulus. One of these levels was chosen for the first trial*. A random number was then generated and compared with the stimulus at the trial level. This random number was regarded as the stimulus which would be necessary to cause a response. In other words it was the critical level of the test item. Therefore if the stimulus at the trial level was equal to or greater than the random number, the result was recorded as a success and the next trial made at the next lower stimulus level. If the result was not a success it was noted as a failure and the next trial made at the next higher stimulus level. This process was continued until the sample was exhausted. This set of trials constitutes one Bruceton test. The process is illustrated in Figures 1 through 4 which show results from Bruceton tests which were part of the investigations described in later paragraphs. Here each line represents a trial level. Each column represents a trial on an individual item. At the bottom of each column the sensitivity and the item number are given. The result of each trial is indicated by an X for success or an O for failure. Since the critical level of each item is known (a situation which does not exist in real life) the mean and standard deviation of the sample can be computed in the usual way. The estimates obtained from the Bruceton test can therefore be compared with the values obtained from the sample as well as with the population parameters. Previous Monte Carlo investigations (see Reference 3 and Figure 5) showed that there was considerable error inherent in estimates of end points as close to the mean as the 10% and 90% points; and even in estimates of the mean, when very small sample sizes were used. We wished to identify the sources and measure the magnitudes of such errors. The investigations described below, while by no means exhaustive, shed considerable light on the problem.

---

*We distinguish between the terms trial, test, and run as follows:

(1) A trial is made on a single item by comparing a random number (representing the sensitivity requirement of the item) with an appropriately selected stimulus level.

(2) A number of these trials collected together and analyzed as a group to estimate parameters of the source population is termed a test.

(3) A group of similar replicate tests carried out usually to assess sampling error is called a run.

BIAS IN ESTIMATION OF STANDARD DEVIATION

Martin has shown[4] that a Bruceton test of fifty items or less has a serious bias in the estimation of the standard deviation. The standard deviation obtained from the Bruceton test tends to be too small. We have checked his results by making Bruceton tests of twenty-five items each and also of two hundred items each. Tables 1 and 2 give the results of these tests. For each Bruceton test the Bruceton estimates of the mean and standard deviation are given together with the values of these parameters for both the sample and the population. Table 1, which gives the results of Bruceton tests of twenty-five trials each, shows that for that sample size the Bruceton test yields a good estimate of the mean (compare averages of columns 1 and 2) but the estimate of the standard deviation is too small (compare averages of columns 3 and 4). Table 1 also shows that in only ten of the fifty tests the Bruceton estimate of the standard deviation was greater than the actual value for the sample. If the Bruceton tests were not biased, i.e., the result of the Bruceton was as likely to be large as to be small, then the probability of seeing only ten of fifty large would be on the order of one in one hundred thousand.

Another approach is to subtract the sample standard deviation from each of the Bruceton standard deviations. The mean of these differences is -0.38* and the standard deviation of this mean is 0.093. If the Bruceton test were not biased the mean difference taken in this way for a set of tests should form a Gaussian distribution about zero. Using the t-test to check this hypothesis we get a value of t equal to 4.09 which is significant at more than the 99.99% level.

We must therefore consider the Bruceton estimate of the standard deviation to be biased when as few as twenty-five items are used for one test. For the case of the two hundred trial Bruceton tests given in Table 2 not only is the mean well estimated but the standard deviation is much better estimated. In this case twenty-four of the sixty-three tests gave standard deviations higher than those of the sample. Also the mean difference, as above, was -0.0251 and the standard deviation of this mean was 0.01543. This gives 1.627 as the value of t, which is significant at the 90% probability level, but not at the 95% level. Summarizing the results we can say that the Bruceton test gives a biased estimate of the standard deviation. This estimate is, on the average, on the order of 20% low for small samples. As a result of this underestimation any prediction of either reliability or safety which makes use of the standard deviation can be expected to be too optimistic. This is one reason that, while Bruceton estimates of the mean are quite good, probability statements about the tails of the distribution must be considered to be unreliable.

*The mean of the differences should be the same as the differences of the means (except for rounding errors). The averages of the third and fourth columns of Table 1 are 1.7170 and 1.3362 and their difference, taken as above, is -0.3808.

VALUES OF G AND H

The values of the standard deviations s_m and s_s of m and s are often required in statistical analysis. These values may be computed from Bruceton tests by the use of Eq's (3) and (4) of Reference 1:

$$\sigma_m = G\sigma/\sqrt{N} \qquad (1)$$

and

$$\sigma_s = H\sigma/\sqrt{N} \qquad (2)$$

where σ, σ_m, and σ_s are population parameters, N is the number of successes or failures (whichever is least) in a single Bruceton test, and G and H are functions of the ratio of the step size to σ and also of the position of the mean with respect to the nearest test level.

In practice we do not know the values of σ, σ_m, and σ_s so we must use their estimates s, s_m, and s_s so that Eq's (1) and (2) become

$$s_m = Gs/\sqrt{N} \qquad (3)$$

and

$$s_s = Hs/\sqrt{N} \quad . \qquad (4)$$

The asymptotic values of G and H (values for an infinite number of trials in a single Bruceton test) are given graphically in Reference 1. As a result of our Monte Carlo tests we have obtained estimates s, s_m, and s_s (Tables 1 and 2). Using these in Eq's (3) and (4) we can compute values of G and H. We can also make a similar computation using σ (of the known distribution) in place of s along with the Monte Carlo values for s_m and s_s. We wish to compare these values of G and H with the asymptotic values obtained from the curve. The values obtained in these different ways will be designated as $G_{i,j,k}$ and $H_{i,j,k}$ where the subscripts i, j, k have the following significance. Values obtained from the Monte Carlo results as just explained are denoted by i = 1. Asymptotic values read from the curves of Reference 1 are denoted by i = 2. Values of 1 or 2 for j refer to tests of 25 or 200 trials each. When the value of s was used to find G or H the subscript k is 1. A value of k = 2 refers to a G or H based on the value of σ rather than s. To aid the reader we tabulate the subscripts and their meanings:

	1	2
i	Monte Carlo result	Asymptotic value
j	25 trials/test	200 trials/test
k	s used	σ used

The different values of G and H are given in the following table. (It must be remembered that the Monte Carlo tests of 25 trials were made with a different step size than those with 200 trials so that the two effects are confounded.)

Value of G and H

i,j,k	1,1,1	2,1,1	1,2,1	2,2,1	1,1,2	2,1,2	1,2,2	2,2,2
G	1.200	0.970	1.089	1.010	0.945	0.950	1.059	1.004
H	1.572	1.51	1.354	1.36	1.235	1.64	1.317	1.38

The group of Bruceton tests of 25 trials each and the group with 200 trials were each divided into half (25, 25 and 31,32) and each half analyzed separately. The upper values of G and H in the following table are obtained from the first half and the lower values from the second half of the tests. (Of course 2,1,2 and 2,2,2 do not depend upon the Monte Carlo results.)

Value of G and H

i,j,k	1,1,1	2,1,1	1,2,1	2,2,1	1,1,2	2,1,2	1,2,2	2,2,2
G	1.194	0.985	0.925	1.008	0.826	0.950	0.888	1.004
	1.203	0.960	1.227	1.006	1.059	0.950	1.208	1.004
H	1.764	1.41	1.122	1.37	1.220	1.64	1.077	1.38
	1.345	1.57	1.534	1.38	1.184	1.64	1.516	1.38

In this investigation we were interested in seeing how the Monte Carlo results differed from the asymptotic values. That is, we wish to compare $G_{1,j,k}$ with $G_{2,j,k}$ and $H_{1,j,k}$ with $H_{2,j,k}$ for the same values of j and k. Since previous work has shown that the Bruceton estimate of the standard deviation is biased, s being too small for small samples, we would expect that the values of G and H would have to be increased to take care of this bias. We do indeed see that $G_{1,1,1}$ is larger than $G_{2,1,1}$ or $G_{1,1,2}$. $H_{1,1,1}$ is clearly larger than $H_{1,1,2}$ but the comparison between $H_{1,1,1}$ and $H_{2,1,1}$ is not clear due to lack of precision in determining their values. The corresponding comparisons for the Bruceton tests of two hundred trials each does not show any difference. This is to be expected since the bias in the estimation of σ by s is not great for tests of two hundred trials.

CORRELATION OF BRUCETON s WITH SAMPLE s

Not only is the estimate of the standard deviation obtained by the Bruceton test biased but it bears little relation to that of the sample being tested when the number of trials is small. Inspection of the results of individual tests in Table 1 will show many cases in which a small value of the Bruceton standard deviation is associated with a large value for the sample and vice-versa. We have computed the correlation coefficient for the estimate of the standard deviation obtained from the Bruceton test with that of the sample. For tests of two hundred trials the coefficient, r, was 0.4773. The scatter of points about the regression line as compared with the total scatter is given by U,* the coefficient of non-determination, $\sqrt{1-r^2}$; in this case 0.878. If we, from previous experience with a certain type of item, have some knowledge of its standard deviation then we can say that a Bruceton test of two hundred items might be expected to reduce the uncertainty now associated with this estimate to 88% of what it was before. This is not a great improvement. For tests of twenty-five trials the correlation coefficient does not differ significantly from zero. This would mean that we learn practically nothing about the standard deviation of the sample by making a Bruceton test of only twenty-five items.

However, this is somewhat too severe a test since all samples were drawn from the same population, an unrealistic situation. The resultant grouping of the sample values of s over a small range causes the above correlation to be smaller than if the sample values of s were more widely distributed. To investigate this point further we ran Monte Carlo Bruceton tests of twenty-five and fifty items per test where each Bruceton test would be performed on a sample taken from a new, randomly chosen population. The 50% points of these individual populations had a mean of 20 and a standard deviation of 0.5. The standard deviations of the populations were allowed to vary about an expected value of 1.0. For each size of Bruceton test (twenty-five or fifty items) two groups of tests were made. One for the "small range" of variation of population standard deviations, and the other for the "large range" situation. Results are given in Table 3 where we can see, for instance, that for the twenty-five trial "small range" case U is still 0.96. Thus we cannot expect to have gained any more information about sample standard deviations from a twenty-five shot Bruceton test. Although these results are better than those obtained with samples from one fixed population they still show that very little gain in knowledge about the standard deviation can be expected from small Bruceton tests.

*A value of r = 1.0 denotes perfect correlation while a value of U = 0.0 denotes perfect determination.

VARIATION IN ESTIMATES OF MEAN AND STANDARD DEVIATION AS A RESULT OF ORDER OF TESTING

A shortcoming which is inherent in any test in which the result of each trial can be classed only as a success or a failure is that the conclusions drawn are dependent upon which items are tested at the several levels. This effect becomes insignificant when a large number of items is used in one test but can be disastrous for small samples. As an example, suppose that we are testing a sample of six items and that the plan calls for testing three items at each of two stimulus levels. Suppose that our sample contains two items which will respond at either level, two which will fail at either level, and two which will fail at the lower level but respond at the higher level. There is now one chance in ten that the results of our test will be a two-thirds response at the lower level and a one-third response at the higher level.

The Monte Carlo technique was used to investigate this effect on Bruceton tests of twenty-five trials. The random number generator was used to obtain one group of twenty-five normally distributed numbers. The group had a mean of 20.0238 and a standard deviation of 1.0018. Then fifty Bruceton tests were made, each one being a different random selection order of the items in this one group. Figures 1 - 4 give examples of these tests. Reference to these figures has already been made in an earlier paragraph. An explanation of the figures is given at that point. Table 4 gives the estimates of the mean and standard deviation obtained from these fifty Bruceton tests. The standard deviation of s found in the series of tests is a measure of the variability caused by selection order alone, and is very nearly as large as that which Reference (1) predicts for large sample size considering sampling variation only. Results of similar tests carried out with larger group sizes are shown in Tables 5 and 6. Intuitively we expected that the selection order effect would be reduced by increasing the sample size. As can be seen from data group A, Table 7, the standard deviation (standard error) of each of the estimates diminishes as the sample size increases. But the error does not disappear. In fact it stands in approximately constant ratio with the corresponding sampling error shown in data group B. The composite standard deviations, data group C, which allowed for both types of error were obtained by combining the variances:

$$s_C = \sqrt{s_1^2 + s_2^2}$$

where s_c is combined error
s_1 is sample error
s_s is selection order error.

Data group D tabulates s_c/s_1 for the individual cases and estimators. There seems to be no systematic trend in these ratios. Hence, it would appear that estimates of the standard deviations of the mean and standard deviation should be increased by about 30% to allow for the selection order effect.

STARTING LEVEL AND STEP SIZE CONSIDERATIONS

The effect of a poor choice of starting level will be appreciable only in case the step size is much smaller than the standard deviation. Otherwise the test levels will very soon reach the fifty per cent point and the only effect is the probable loss of one or two trials used in getting to the optimum region. However, if the steps are small so that the true fifty per cent point is, say, as much as five steps distant from the starting point we may expect that several reversals will occur before the fifty per cent region is approached. In extreme cases the test may never reach this point. In any case the occasional reversals in the early part of the test will unduly influence the estimates of the mean and standard deviation.

Four Monte Carlo runs were made to obtain some idea of the amount of this effect. Items were selected from a population which had a mean of 20.00 and a standard deviation of 5.00. The step size for the Bruceton tests was 1.0 which is one-fifth of the standard deviation. Fifty Bruceton tests were made in each Monte Carlo run. A run was made for both twenty-five and fifty item Bruceton tests and for starting points of both one and two standard deviations above the mean. The results are summarized in Table 8.

It can be seen that, independently of whether we start at one or two standard deviations above the mean, the final estimates of the mean are also above the mean by an amount which seems to be controlled by the sample size. The larger the sample size, the smaller the bias of the mean and the smaller the variability of the mean, s_m.

Both the estimates of the standard deviation, s, and the spread of these estimates, s_s, are better for the longer runs than for the shorter runs, a predictable trend. For a one-standard deviation starting point the estimates of s are low,

while for this more remote starting point the estimates are too high. This is due to several Bruceton tests which gave very large values for the standard deviation. From Table 9,which lists the results of the individual tests of each run, we can see that the distribution of s is quite skewed for the small step, remote starting level case.

Bruceton tests of this sort,in which they step size is small and the starting point is several steps away from the mean, will, in general, tend to wander toward the fifty per cent point in an irregular fashion. On the way there they will probably pause now and then,and may even reverse the direction of the trend for a short time. A short test may not have time to reach the true fifty per cent region. A longer test would certainly be more likely to do so. The result will be that the short test will give an estimate of the fifty per cent point which is more affected by the starting level than the estimate for a longer test.

SUMMARY

We have discussed five features of short Bruceton tests. First, we have shown that there is a serious bias* in the estimation of the standard deviation which makes estimates of reliability and safety based on short Bruceton tests too optimistic. We have indicated that this bias, though considerably reduced, still is present in tests with as many as two hundred items. Second, we have shown that there is a bias in G(we suspect a bias in H as well even though the results are not as clear cut). A much more exhaustive study would be necessary to identify and measure the biases in s, G,and H. Particularly in the light of the third feature of our study we see little point in pursuing this effort further. Third, we have shown that,because of the poor correlation between the sample s and the Bruceton s, a test of twenty-five items does not afford a much better estimate of the standard deviation than would be obtained from general previous experience with similar items. This is true even if a correction could be made for the known bias of the test. Fourth, we have pointed out that the effect of the selection order in which individual items of the sample are chosen, although negligible in large samples, becomes quite appreciable in a sample of twenty-five. Consequently the standard deviations of both mean and standard deviation are seriously underestimated if the large sample formulas for these quantities are used. Fifth, we have pointed out the effect of a poor choice of starting level* Since this is important only when the stepis quite small it can be avoided by using a larger step size. If this does not seem desirable it would be better to discard all trials made before the test appears to have reached the fifty per cent region. However, this appearance can be deceiving and this choice should, therefore, be avoided. It was pointed out in the

*Ref. 5 reports Monte Carlo results during the development of a Bruceton method applicable to the logistic distribution function, wherein simi.ar bias of the standard deviation and effects of choice of starting level were noted.

discussion that the test may never actually reach the fifty per cent region although it might appear to do so.

REFERENCES

1. Dixon, W. J. and Mood, A. M., "A Method for Obtaining and Analyzing Sensitivity Data", J.Am.Stat.Assoc., 43, pp. 109-126 (1948).

2. Juncosa, M. L., "Random Number Generation on the BRL High-Speed Computing Machines", BRL Report 855, Aberdeen Proving Ground, Md., (1963).

3. NOLTR 63-133, "The Prediction of Very Low EED Firing Probabilities", J. N. Ayres, L. D. Hampton, I. Kabik, Sep 1963.

4. Martin, J. W. and Saunders, J., "Bruceton Tests: Results of a Computer Study on Small Sample Accuracy", International Conference on Sensitivity and Hazards of Explosives", London, 1-3 Oct 1963.

5. NOLTR 66-129, "Bruceton Analysis for Large and Small Samples Using the Logistic Distribution Function", L. D. Hampton, G. Blum, (in preparation).

TABLE 1

RESULTS OF FIFTY
MONTE CARLO BRUCETON TESTS, 25 TRIALS

Mu is 20.0, Sigma is 1.7, 99% Pt is 23.954

MEAN Sample	MEAN BCTN	STD. DEVIATION Sample	STD. DEVIATION BCTN	BCTN 99% Pt
20.0007	19.5000	1.6920	0.5923	20.8778
20.3472	20.3333	1.6765	0.5476	21.6071
20.0057	20.0000	1.5915	1.5309	23.5609
20.3297	20.3333	1.7922	0.8158	22.2309
19.6752	19.6818	1.6180	0.5879	21.0493
20.1317	20.0833	1.8652	0.9834	22.3708
20.1919	19.7500	1.7358	1.4304	23.0770
20.8221	20.9545	2.2485	1.6251	24.7346
19.5097	19.5000	1.6045	1.1287	22.1253
19.9428	20.4167	1.5472	0.4471	21.4566
19.4530	19.9167	1.9248	0.7153	21.5803
20.1272	20.4167	1.6120	0.9834	22.7041
20.3900	20.5833	1.5730	0.7152	22.2470
19.9447	20.5833	1.9408	0.7152	22.2470
20.2268	20.3333	1.4602	1.6203	24.1022
19.3768	19.8333	1.9447	0.6817	21.4190
19.8524	20.0000	2.0298	1.2627	22.9372
20.2610	19.6667	1.9460	1.8885	24.0593
20.5070	20.5833	1.8740	2.3242	25.9895
20.2091	20.0833	2.5638	1.5197	23.6183
19.9338	19.6667	1.3919	0.8158	21.5643
19.7454	20.0000	1.6347	0.9946	22.3134
20.4902	20.4167	1.7512	1.2516	23.3279
20.2244	20.3333	1.5923	2.9611	27.2209
19.8095	19.4167	1.2813	1.2516	22.3279
19.5360	19.3333	1.9596	1.8885	23.7259
20.1724	20.5833	1.7299	0.9834	22.8708
20.0883	20.3333	1.4376	1.6203	24.1022
19.8341	20.1667	1.5012	1.2181	22.9999
20.4485	20.4167	1.6283	1.2516	23.3279
20.3624	20.7500	1.4482	1.1622	23.4532
19.9142	19.4167	1.5153	1.2516	22.3279
19.4667	19.3333	1.6649	2.1566	24.3497
19.8363	20.0000	1.5260	0.9946	22.3134
19.6948	20.2500	1.5706	1.1622	22.9532
19.5888	20.0833	2.4564	0.9834	22.3708
19.6142	19.4167	1.7379	1.5198	22.9516
20.0624	20.1667	1.3766	1.4862	23.6236
20.0638	20.6667	1.6661	1.3521	23.8118

Continued on next page

TABLE 1 CON'T

	MEAN Sample	MEAN BCTN	STD DEVIATION Sample	STD DEVIATION BCTN	BCTN 99% Pt
	19.9526	20.0833	1.9287	1.7879	24.2420
	19.5119	19.3333	1.9598	0.8158	21.2309
	19.9759	20.5833	1.4984	1.2516	23.4945
	20.2410	20.0000	1.5280	0.9946	22.3134
	20.4971	20.7500	1.5391	1.9667	25.3245
	19.9567	19.4091	1.8489	1.6517	23.2510
	20.4976	20.5833	2.0324	1.7879	24.7420
	19.7781	19.6667	1.5482	2.1566	24.6830
	19.7283	19.3333	1.3067	1.6203	23.1022
	20.1219	20.6667	1.8365	0.8158	22.5643
	20.1679	20.5000	1.7138	3.5422	28.7391
Average	20.0124	20.0842	1.7170	1.3362=s	
Std.Dev.	0.3292	0.4615=s_m	0.2639	0.6062=s_s	

Notes:

Mu = μ = population mean
Sigma = σ = population standard deviation
99% Pt. = stimulus needed for 99% response as computed from mean and standard deviation
Sample = individual true sample value
BCTN = Bruceton estimate

TABLE 2

RESULTS OF SIXTY-THREE
MONTE CARLO BRUCETON TESTS, 200 TRIALS

Mu is 20.5, Sigma is 1.00, 99% Pt is 22.826

MEAN Sample	MEAN BCTN	STD DEVIATION Sample	STD DEVIATION BCTN	BCTN 99% Pt
20.4804	20.5000	0.9641	1.1179	23.1003
20.5797	20.5303	1.0457	1.1109	23.1143
20.4927	20.5300	0.9936	1.0039	22.8650
20.5542	20.6000	0.9847	0.8766	22.6390
20.5433	20.4300	1.1079	1.0296	22.8249
20.3367	20.3081	0.8939	0.8906	22.3797
20.4380	20.4600	0.9993	1.1476	23.1292
20.5235	20.5100	1.0319	0.9408	22.6983
20.5228	20.4700	1.0525	1.0682	22.9547
20.6243	20.7600	0.9060	0.8805	22.8079
20.5528	20.4600	0.9440	0.7614	22.2310
20.5359	20.5200	0.9844	0.9564	22.7446
20.5580	20.5600	0.9641	1.0800	23.0720
20.4433	20.5202	1.0213	1.0630	22.9930
20.4167	20.3600	1.0125	0.8612	22.3630
20.4140	20.5400	1.0607	1.0510	22.9845
20.4508	20.5000	0.9261	0.8927	22.5764
20.4015	20.4800	1.0896	1.0530	22.9292
20.4528	20.5100	0.9239	0.8764	22.5486
20.4207	20.3600	1.0231	0.8933	22.2736
20.5113	20.5300	0.9327	1.0039	22.8650
20.5178	20.4700	0.9980	1.0039	22.8050
20.4698	20.4697	0.9810	0.8184	22.3733
20.4497	20.3282	0.9705	0.9675	22.5786
20.4166	20.4100	0.9782	0.7348	22.1192
20.4664	20.6300	1.0725	1.0103	22.9800
20.4506	20.5100	1.0656	0.8764	22.5486
20.4989	20.4100	0.9178	0.7992	22.2689
20.3767	20.4100	0.9881	0.9279	22.5684
20.4948	20.5101	1.0029	1.0147	22.8705
20.4789	20.4600	1.0225	1.0510	22.9047
20.4189	20.4300	1.0083	1.1583	23.1243
20.4232	20.4300	1.0222	1.1583	23.1243
20.4054	20.3485	1.0197	1.1405	23.0013
20.3380	20.3182	1.0021	0.9455	22.5173
20.4107	20.4100	1.0032	0.8957	22.4935
20.4996	20.3300	1.0068	1.1841	23.0842
20.4831	20.4800	0.9090	0.8599	22.4801
20.4336	20.3900	0.9982	0.9215	22.5334
20.3863	20.3687	1.0032	0.8896	22.4380
20.5004	20.4300	0.9906	0.9974	22.7500

CONTINUED ON NEXT PAGE

TABLE 2 CON'T

	MEAN		STD. DEVIATION		BCTN
	Sample	BCTN	Sample	BCTN	99% Pt
	20.4057	20.3800	0.9928	0.9339	22.5522
	20.5650	20.6000	0.9986	1.1984	22.3875
	20.4689	20.4300	1.0151	1.0296	22.8249
	20.5007	20.6300	1.0762	1.1712	23.3542
	20.4812	20.5101	0.9360	1.0147	22.8704
	20.7165	20.7200	0.9518	1.0723	22.2141
	20.4453	20.4500	0.8752	0.8726	22.4796
	20.4336	20.4100	1.0717	0.8636	22.4186
	20.4573	20.4900	1.0097	1.1661	23.2023
	20.5064	20.4400	1.0819	0.9834	22.1127
	20.3943	20.4700	1.0698	1.0039	22.8039
	20.4484	20.4200	1.0508	1.3973	23.6700
	20.6071	20.7424	1.0028	0.6766	22.3161
	20.4211	20.3384	1.0314	0.8916	22.4123
	20.5006	20.5700	0.9679	0.8043	22.4409
	20.4403	20.3800	0.8816	0.7408	22.1031
	20.4803	20.3687	0.8927	0.9672	22.6648
	20.5425	20.5300	1.0158	0.9073	22.6404
	20.6106	20.7121	1.0497	0.9100	22.8288
	20.4243	20.4700	1.0111	0.8430	22.4308
	20.5899	20.6600	1.0962	0.9802	22.9400
	20.5355	20.6500	0.9594	0.9369	22.8293
Average	20.4785	20.4818	0.9978	0.9727 $= s$	
Std.Dev.	0.0711	0.1059 $= s_m$	0.0552	0.1317 $= s_s$	

Notes:

Mu = μ = population mean
Sigma = σ = population standard deviation
99% Pt. = stimulus needed for 99% response
Sample = individual true sample value
BCTN = Bruceton estimate

TABLE 3

CORRELATION OF BRUCETON s WITH SAMPLE s

No.of Trials	25	50
Small Range 0.492<s<2.038	r = 0.2837 U = 0.96	r = 0.6202 U = 0.78
Large Range 0.258<s<3.341	r = 0.6212 U = 0.78	r = 0.8657 U = 0.50

r = correlation coefficient

U = coefficient of non-determination

TABLE 4

TWENTY-FIVE TRIAL SELECTION ORDER EXPERIMENTS

Sample mean 20.0238, Standard deviation 1.0018

Mean	Std. Dev.	99% Pt.
0.2500	1.4313	23.5792
0.0833	0.9805	22.3639
0.1667	0.9467	22.3686
9.7500	0.3493	20.5625
0.0833	0.9805	22.3639
9.7500	0.8903	21.8209
0.0833	0.7100	21.7348
9.6669	0.5409	20.9249
0.0000	0.7212	21.6776
0.2500	1.4313	23.5792
0.0833	1.2510	22.9931
9.6667	0.8114	21.5540
0.0000	0.7212	21.6776
9.9167	0.7100	21.5681
0.0833	0.9805	22.3639
0.0000	1.5327	23.5652
0.1667	0.4057	21.1103
0.1667	0.6762	21.7395
9.9167	0.9805	22.1973
0.0833	1.2510	22.9931
0.0000	0.9917	22.3068
0.0000	0.9917	22.3068
9.8333	0.6762	21.4061
9.7500	0.8903	21.8209
0.0000	1.2622	22.9360
0.1667	0.9467	22.3686
9.6667	0.5409	20.9249
9.8333	0.6762	21.4061
0.1667	1.2172	22.9978
.9167	1.2510	22.8265
.8333	1.2172	22.6645
.3333	0.8114	22.2207
.1667	0.9467	22.3686
.7500	0.8903	21.8209
.000	0.9917	22.3068
.667	0.5409	20.9249
.667	1.0819	22.1832
.0833	0.4395	21.1056
.0833	0.7100	21.7348
.2500	0.8903	22.3209

Mean	Std. Dev.	99% Pt.
20.3333	1.0819	22.849
20.3333	1.6229	24.1087
19.8333	0.9467	22.0353
19.9167	0.9805	22.1973
19.7500	1.1608	22.4500
20.0833	0.4395	21.1056
19.5833	1.2510	22.4931
19.8333	0.9467	22.0353
20.0833	0.9805	22.3639
20.0000	1.5327	23.5652

TABLE 5

ONE-HUNDRED TRIAL SELECTION ORDER EXPERIMENTS

Sample mean 20.0285, Standard deviation 0.8908

Mean	Std. Dev.	99% Pt
19.9490	0.6453	21.4499
19.8878	0.6953	21.5050
20.0600	0.9463	22.3029
20.0800	0.5702	21.4063
20.0800	0.8948	22.1613
20.0800	0.8299	22.0103
19.9200	0.9597	22.1523
20.0200	0.6449	21.5200
20.1000	0.7591	21.8657
19.9800	0.8396	21.9330
19.8600	0.8085	21.7405
19.8800	0.9467	22.0821
19.9000	0.7591	21.6657
19.9400	0.8994	22.0319
19.8200	0.7877	21.6522
20.0000	0.7104	21.6525
20.0400	0.7078	21.6864
20.0600	0.7695	21.8499
19.9286	0.8399	21.8823
20.0400	0.9675	22.2904
20.0000	0.9052	22.1055
20.0510	0.8440	22.0142
20.0800	0.9597	22.3123
20.1200	0.8169	22.0201
20.0800	0.7000	21.7083
20.0102	0.5168	21.2124
20.0200	0.9695	22.2750
20.0400	1.1623	22.7435
19.9000	0.6293	21.3637
20.0800	1.1545	22.7653
20.0200	1.0344	22.4260
20.0400	0.9675	22.2904
19.9800	0.7747	21.7820
20.0200	0.7747	21.8220
20.0600	0.5098	21.2459
19.8200	0.5280	21.0481
20.1000	0.8240	22.0167
20.2200	0.8916	22.2938
19.9200	0.7000	21.5483
19.9898	0.7818	21.8083
20.0800	1.0246	22.4633

Mean	Std. Dev.	99% Pt
20.1000	1.0246	22.4633
19.9490	0.7115	21.6040
19.9600	0.8377	21.9085
20.0918	0.7020	21.7248
20.1000	0.6942	21.7147
20.1400	1.0032	22.4735
20.1000	0.6942	21.7147
19.9800	0.8396	21.9330
19.9800	0.8396	21.9330

TABLE 6

TWO-HUNDRED TRIAL SELECTION ORDER EXPERIMENTS

Sample mean 19.9953, Standard deviation 0.9866

Mean	Std. Dev.	99% Pt
20.0000	1.0675	22.4830
19.9000	0.9214	22.0432
19.9300	0.8972	22.0170
19.9800	0.8072	21.8575
19.9500	1.0634	22.4236
20.1364	0.9452	22.3349
19.9300	1.1245	22.5455
20.0800	0.9273	22.2368
20.0400	0.9351	22.2149
20.0100	0.9375	22.1906
19.9747	1.0071	22.3173
19.9300	0.9297	22.0925
19.9000	0.8565	21.8922
20.0152	0.9094	22.1304
19.8300	0.9881	22.1284
20.1100	0.9505	22.3208
19.9444	0.8392	21.8964
19.8600	1.0032	22.1935
19.9141	0.7339	21.6212
19.9500	0.8687	21.9706
19.9900	1.1323	22.6236
19.9800	1.0344	22.3860
19.9100	0.7947	21.7584
19.8600	1.0032	22.1935
19.9242	0.9333	22.0950
19.9400	0.8994	22.0319
19.9700	0,8064	21.8456
19.9900	1.1647	22.6991
20.0455	1.1359	22.6877
20.0300	0.7090	21.6791
20.0500	0.9336	22.2216
19.9200	0.9922	22.2278
20.0200	0.8721	22.0485
19.9300	1.1245	22.5455
19.9800	1.0344	22.3860
20.0500	0.9336	22.2216
19.9400	0.8994	22.0319
19.9900	0.8401	21.9441
20.0100	1.0024	22.3416
19.9747	1.0399	22.3936
19.8600	0.8085	21.7405

Mean	Std. Dev.	99% Pt
20.0900	0.9245	22.2404
20.0800	1.0896	22.6143
19.9400	0.9318	22.1074
19.9700	0.9037	22.0721
19.8600	0.8085	21.7405
20.1100	0.9505	22.3208
20.0000	1.2298	22.8605
20.0100	1.0673	22.4926
20.0000	1.0350	22.4075

TABLE 7

STANDARD DEVIATIONS AND THEIR CORRECTION TO ACCOUNT FOR SELECTION ORDER EFFECT

Sample Size	Mean	s	99% Pt.
25	0.198	0.3038	0.7806
100	0.0857	0.1541	0.3795
200	0.0705	0.1132	0.2083
A: Variation due to order alone.			
25	0.289	0.4046	0.9843
100	0.1414	0.197	0.4802
200	0.100	0.1414	0.3406
B: Sampling variation as in Reference (1)			
25	0.3503	0.5060	1.2562
100	0.1653	0.2501	0.6121
200	0.1224	0.1811	0.3992
C: Combined standard deviation allowing for both selection order and sampling variation.			
25	1.212	1.251	1.276
100	1.169	1.270	1.275
200	1.224	1.281	1.172
D: Correction Factor, $\frac{C}{B}$			

TABLE 8

EFFECT OF POOR CHOICE OF STARTING LEVEL ON ESTIMATED PARAMETERS

Mu = 20.0, Sigma = 5.0

Items per test	25	50	25	50
Starting level	25	25	30	30
m	21.179	20.522	21.251	20.458
s_m	1.274	0.909	1.767	1.170
s	3.512	4.683	7.458	7.308
s_s	3.054	2.191	8.181	4.740

TABLE 9A

INDIVIDUAL TEST RESULTS FOR INVESTIGATION
OF EFFECT OF STARTING LEVEL

Mu = 20.0; Sigma = 5.0; Starting Level = 25.0; 25 Trials

Mean	Std.Dev.	Mean	Std.Dev.
19.318	2.632	19.400	5.977
21.318	5.547	20.200	1.360
20.800	4.566	20.591	1.943
21.045	1.042	19.300	1.921
20.000	1.264	21.591	4.858
21.900	3.332	23.500	3.909
23.944	6.870	22.833	1.754
19.278	7.464	21.864	1.016
21.833	7.899	20.500	2.977
20.389	7.345	22.333	1.086
20.136	4.513	20.800	2.963
20.833	9.235	20.722	2.120
22.071	0.455	20.100	2.050
21.500	4.725	21.400	2.130
22.100	3.012	22.250	3.034
19.500	2.306	19.500	2.199
22.833	1.220	19.250	19.198
22.100	3.012	20.227	1.837
21.773	3.003	22.167	3.624
23.100	1.088	23.300	3.845
21.500	2.685	21.136	1.307
22.864	3.347	18.900	1.088
21.227	1.254	20.611	5.920
23.056	3.664	20.056	2.239
20.389	2.714	21.591	1.069

TABLE 9B

INDIVIDUAL TEST RESULTS FOR INVESTIGATION
OF EFFECT OF STARTING LEVEL

Mu = 20.0; Sigma = 5.0; Starting Level = 25.0; 50 Trials

Mean	Std.Dev.	Mean	Std.Dev.
20.250	3.969	22.167	6.697
20.326	6.426	20.227	3.732
19.045	3.811	20.364	4.768
19.891	1.420	21.136	3.639
20.909	4.530	21.674	5.310
21.227	4.606	21.239	4.413
19.370	2.613	20.909	7.153
20.545	4.066	21.717	1.450
21.136	5.533	22.891	7.692
21.591	7.189	19.000	7.458
21.500	4.422	21.000	10.664
20.400	2.931	20.045	6.289
19.909	5.696	19.239	2.601
20.022	4.226	19.136	3.202
21.227	4.898	19.375	6.115
18.955	8.620	20.630	2.892
20.409	5.878	21.065	5.753
21.273	5.153	20.958	2.864
19.591	2.089	19.717	2.145
20.452	6.852	19.881	2.883
20.674	1.407	21.717	3.401
21.848	2.238	21.500	4.383
20.227	3.440	19.595	3.101
20.583	11.405	19.864	3.056
19.413	5.207	20.292	3.800

TABLE 9C

INDIVIDUAL TEST RESULTS FOR INVESTIGATION OF EFFECT OF STARTING LEVEL

Mu = 20.0; Sigma = 5.0; Starting Level = 30.0; 25 Trials

Mean	Std.Dev.	Mean	Std.Dev.
19.071	1.371	22.900	26.095
20.357	0.716	22.167	15.914
23.625	7.451	23.000	7.516
19.000	29.317	22.833	1.843
20.357	9.876	23.500	2.466
19.667	1.353	22.750	3.168
19.750	3.969	21.500	1.487
22.786	12.755	21.375	7.451
21.375	10.657	22.500	6.474
22.625	6.249	22.500	0.863
20.333	6.162	26.375	5.848
20.786	4.511	20.125	5.247
22.167	1.131	18.333	1.353
19.167	14.311	21.375	1.039
20.500	9.222	25.643	3.006
19.667	37.154	21.167	2.021
21.500	7.275	18.786	1.305
21.500	1.264	21.000	30.385
19.786	3.137	22.500	16.092
20.375	0.638	22.214	7.717
17.929	6.409	18.000	0.463
21.750	11.183	22.071	9.615
19.333	2.956	20.214	3.137
22.667	8.834	20.500	1.665
22.250	11.183	20.875	1.640

TABLE 9D

INDIVIDUAL TEST RESULTS FOR INVESTIGATION
OF EFFECT OF STARTING LEVEL

Mu = 20.0; Sigma = 5.0; Starting Level = 30.0; 50 Trials

Mean	Std.Dev.	Mean	Std.Dev.
19.850	1.228	20.595	8.139
21.250	7.897	21.071	8.088
21.273	3.258	21.868	5.328
20.595	13.329	18.650	5.236
19.444	24.369	20.310	3.668
21.350	10.686	19.711	15.346
21.214	17.488	21.000	13.607
21.643	5.907	21.976	7.637
22.050	7.672	19.605	3.588
19.773	7.229	21.650	13.251
18.500	7.318	19.200	3.284
19.950	5.107	19.950	3.184
21.950	3.825	20.132	2.291
19.447	3.179	20.342	11.074
19.650	7.961	20.405	4.475
20.816	3.446	19.405	2.643
19.250	2.607	21.450	2.703
21.119	8.226	23.738	6.154
18.900	4.454	19.667	10.793
19.250	8.538	20.100	8.141
21.400	6.137	21.868	4.653
19.395	5.106	20.816	8.339
22.658	10.399	20.395	4.769
21.200	5.208	20.000	19.521
19.350	2.671	18.700	6.250

m = 20.2500 s = 1.4313

STIMULUS LEVEL	22					X										X								X			3	0
	21				O		X								O		X		X				O		X		4	3
	20	X		O				X		X				O				O		X		O				O	4	5
	19		O						O		X		O								O						1	4
	18											O															0	1
																											FIRE	FAIL
CRITICAL SENSITIVITY		19.65	19.47	20.86	21.22	19.38	18.57	19.72	21.21	19.20	18.73	19.20	19.69	20.25	21.27	18.37	20.64	20.80	18.57	19.05	21.41	20.66	21.31	19.47	20.97	20.94		
ITEM NO.		15	12	20	19	22	7	24	13	11	8	16	6	21	3	10	5	2	9	25	23	4	14	18	1	17		

FIG. 1 BRUCETON TEST NO. 1

m = 20.0833 s = 0.9805

STIMULUS LEVEL	22															X											1	0
	21		X						X				X		O		X						X		X		6	1
	20	O		X		X		O		X		O		O				X				O		O		X	5	6
	19				O		O				O								X		O						1	4
	18																			O							0	1
																											FIRE	FAIL
CRITICAL SENSITIVITY		20.66	19.65	19.72	19.05	19.69	21.41	20.25	20.94	19.20	19.20	20.94	18.73	21.27	21.31	19.47	18.37	18.57	18.57	21.21	19.47	21.22	20.80	20.64	20.86	19.38		
ITEM NO.		4	15	24	25	6	23	21	1	11	16	17	8	3	14	18	10	9	7	13	12	19	2	5	20	22		

FIG. 2 BRUCETON TEST NO. 2

M = 20.1667 S = 0.9467

STIMULUS LEVEL																											
22															X										X	2	0
21				X								X		O		X				X				O		4	2
20	X		O		X		X		X		O		O				X		O		X		O			6	5
19		O				O		O		O								O				O				0	6
18																										FIRE	FAIL
CRITICAL SENSITIVITY	18.73	19.72	20.80	19.65	19.05	21.22	19.38	19.47	19.47	21.21	20.94	19.20	20.25	21.27	18.57	18.37	18.57	20.94	21.41	20.86	19.20	20.66	20.64	21.31	19.69		
ITEM NO.	8	24	2	15	25	19	22	12	18	13	17	16	21	3	7	10	9	1	23	20	11	4	5	14	6		

FIG.3 BRUCETON TEST NO. 3

M = 19.7500 S = 0.3493

STIMULUS LEVEL																											
21		X																		X		X				3	0
20	O		X		X		X		X		X		X		X		X		O		O		X		O	9	4
19				O		O		O		O		O		O		O		O						O		0	9
																										FIRE	FAIL
CRITICAL SENSITIVITY	20.86	20.80	19.47	20.64	19.20	19.72	19.69	19.47	18.57	20.94	18.57	20.25	18.73	21.31	19.38	21.41	19.05	21.27	20.66	18.37	21.21	19.65	19.20	20.94	21.22		
ITEM NO.	20	2	12	5	16	24	6	18	9	17	7	21	8	14	22	23	25	3	4	10	13	15	11	1	19		

FIG. 4 BRUCETON TEST NO. 4

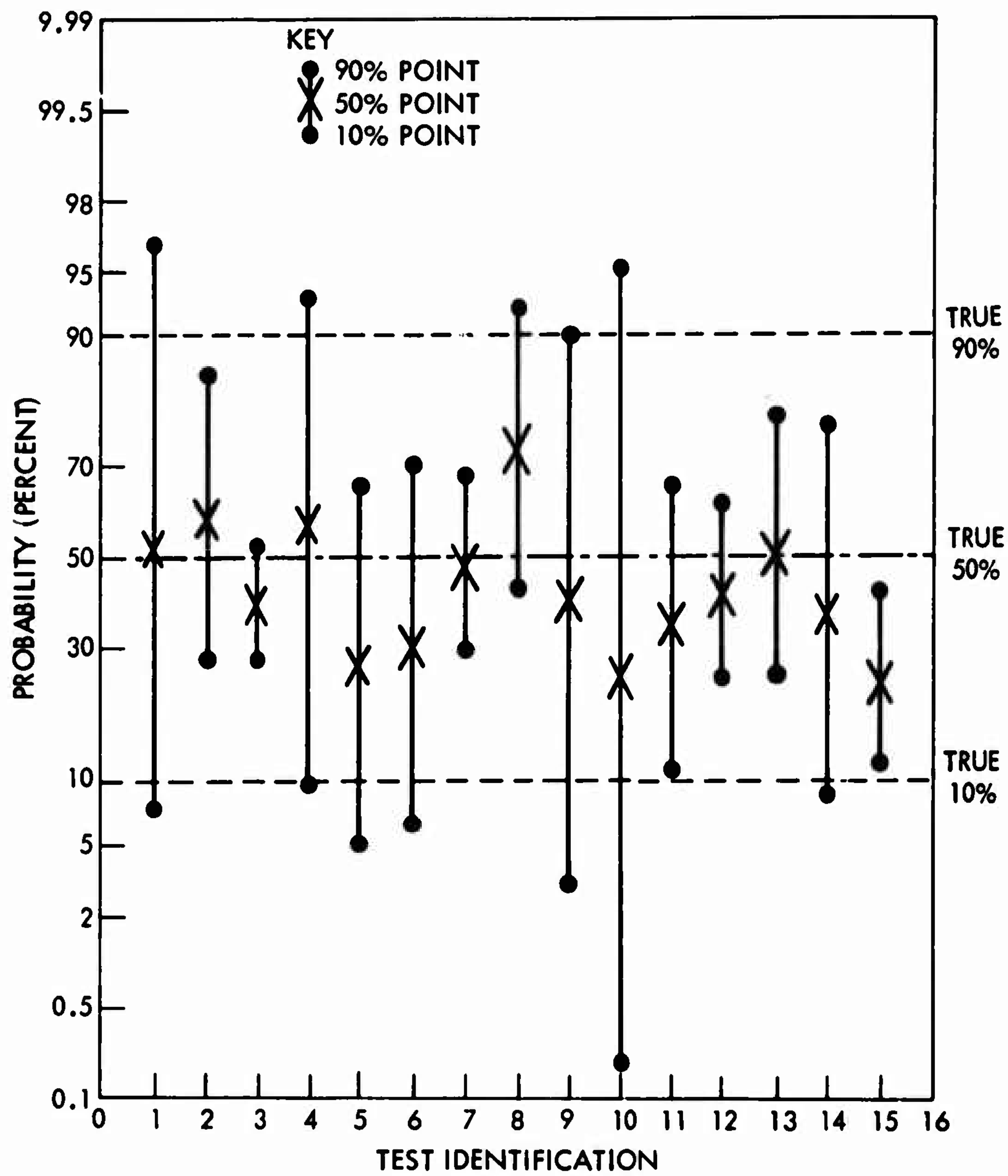

FIG.5 VARIABILITY OF ESTIMATES OF 90, 50, AND 10% POINTS BASED ON 20-SHOT BRUCETON SAMPLES FROM A KNOWN POPULATION

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U. S. Naval Ordnance Laboratory White Oak, Md.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP --

3. REPORT TITLE

MONTE CARLO INVESTIGATIONS OF SMALL SAMPLE BRUCETON TESTS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

--

5. AUTHOR(S) (First name, middle initial, last name)

L. D. Hampton

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
17 February 1967	24	5

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. Task NOL 443/NWL. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NOLTR 66-117 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES --	12. SPONSORING MILITARY ACTIVITY U. S. Naval Weapons Laboratory Dahlgren, Va.

13. ABSTRACT

Monte Carlo investigations of Bruceton tests (twenty-five and fifty trials) show the following characteristics: (1) a bias in the estimate of the standard deviation causes predictions of reliability or safety based on such tests to be too optimistic; (2) there are additional biases (in the parameters G and H) which will cause underestimates of the errors of the mean and standard deviation; (3) there is little correlation between the actual standard deviation and its estimate as obtained from the short Bruceton test; (4) the order in which the items of the sample are tested has a serious effect upon the estimates obtained; (5) a poor choice of starting level can give misleading results when the step is small.

DD FORM 1473, 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

UNCLASSIFIED
Security Classification

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
Safety						
Test						
Bruceton						
Explosive						
Monte Carlo						
Standard Deviation (Sigma)						
Sample Size						
Order						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

UNCLASSIFIED
Security Classification